TRINITY COLLEGE

IN NOMINE DOMINI

MATHEMATICS SPECIALIST UNITS 3&4

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

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In words _____

_____

Your name _____

Number of additional answer booklets used (if applicable): ⬚

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	48	35
Section Two: Calculator-assumed	13	13	100	90	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed   **65% (90 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9   **(6 marks)**

Let u, v and w be complex numbers, and r and θ be constants so that

$$|u| = r, \qquad \arg(u) = \theta, \qquad v = (\sqrt{3} - i)u, \qquad w = \frac{v}{1 + i}.$$

Determine the following in terms of r and / or θ.

(a) $\text{Arg}(w)$.   (3 marks)

(b) $|w^2|$.   (3 marks)

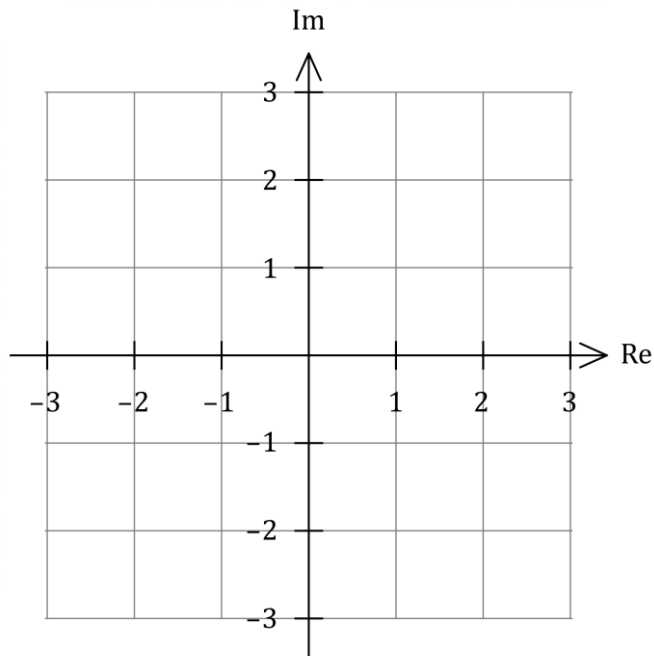**Question 10**   **(7 marks)**

Let R be the region of the complex plane where the inequalities $|z - 1 - i| \leq 1$ and $|z + \bar{z}| \geq 3$ hold simultaneously.

(a)   Sketch R on the axes below.   (5 marks)

(b)   Determine the maximum value of $\mathrm{Im}(z)$ in R.   (2 marks)

Question 11   **(7 marks)**

In order to estimate the mean cost of damage sustained by parked vehicles when struck by another vehicle, an insurance company examined the records of 75 such occurrences, and obtained a sample mean of $3361 with sample standard deviation $326.

(a)   Construct a 95% confidence interval for the mean cost of damage in all such accidents.

     (3 marks)

(b)   Previously, the insurance company had used the amount of $3290 for the mean cost of damage in all such accidents. State, with reasons, whether this amount is no longer valid.
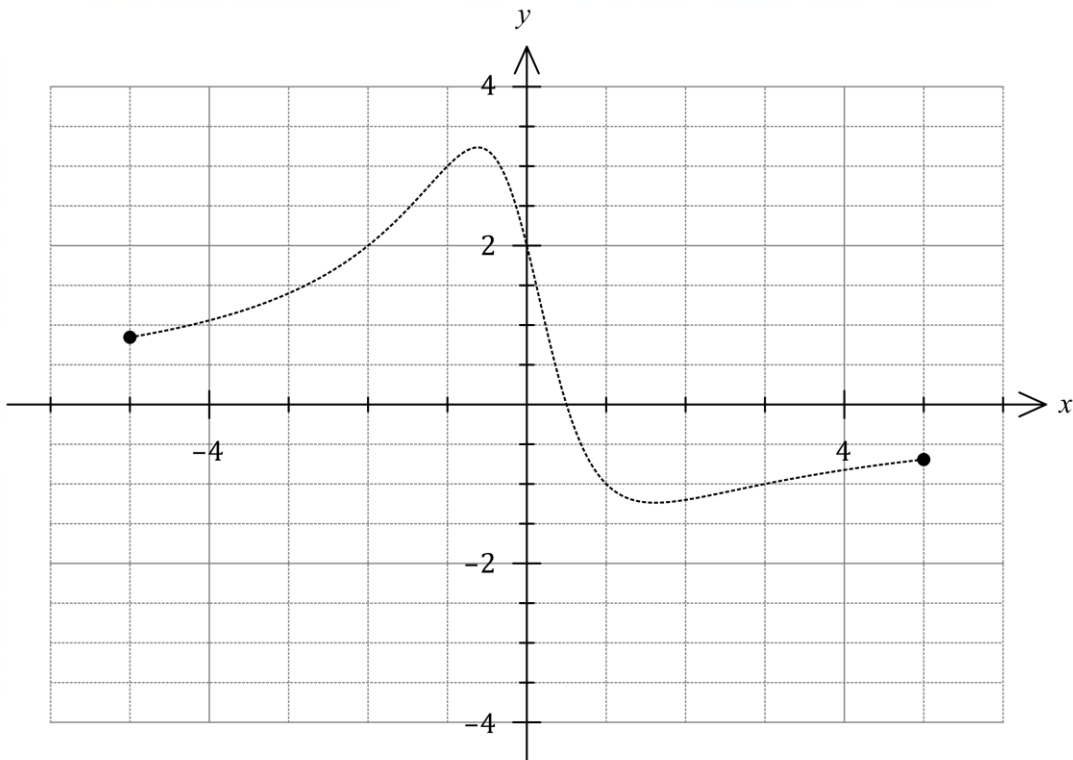
     (2 marks)

(c)   State one assumption made in constructing the interval in part (a) and comment on how reasonable this assumption is in relation to the information provided.   (2 marks)
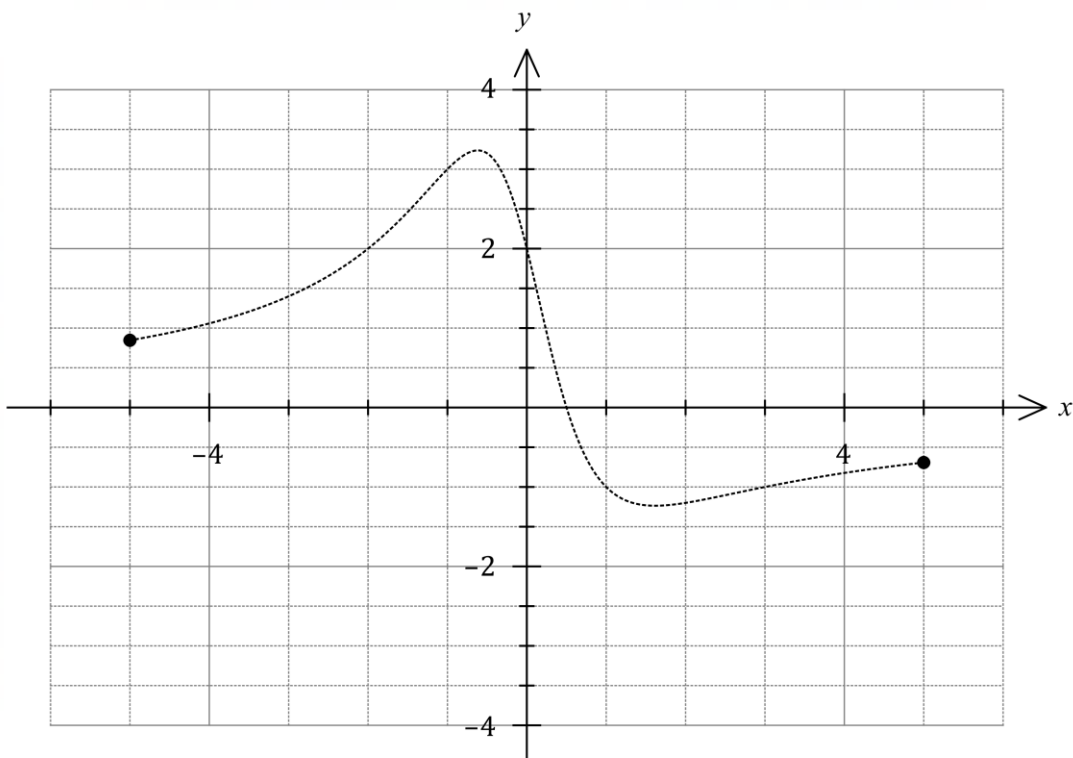
Question 12 (6 marks)

(a) The dotted curve on the axes below is the graph of $y = f(x)$. On the same axes, sketch the graph of $y = \dfrac{1}{f(x)}$. (4 marks)

(b) The dotted curve on the axes below is the graph of $y = f(x)$. On the same axes, sketch the graph of $y = -|f(x)|$. (2 marks)

See next page

Question 13 **(8 marks)**

A person who weighs 108 kg begins a specialist diet so that their rate of weight loss can be modelled by

$$\frac{dw}{dt} = k(w - 76)$$

where w is the persons weight in kilograms and t is the number of days since the diet began.

After 1 week the person had lost a total of 5.7 kg.

(a) Show use of the separation of variables method to obtain a function for w in terms of t.

(5 marks)

(b) At what rate is the person losing weight after 3 weeks? (2 marks)

(c) State the total weight that this person is expected to lose if they maintain the diet.

(1 mark)

Question 14   **(8 marks)**

The coordinates of the three vertices of a triangle are $A(-2, 1, 3)$, $B(-1, 0, 5)$ and $C(1, 2, 2)$.

(a)   Prove that the triangle is right-angled at A.   (2 marks)

(b)   Determine the Cartesian equation of plane that contains the triangle.   (3 marks)

(c)   Determine the exact vector equation of the sphere that has diameter BC.   (3 marks)

Question 15   **(7 marks)**

The mass of raisins in each 750 g packet of muesli produced by a company is normally distributed with a mean of 54.5 g and standard deviation 4.4 g.

(a)   Determine the probability that the total mass of raisins in a random sample of 56 packets of muesli is at least 3080 g.   (4 marks)

(b)   Another random sample of packets is to be taken. Determine the minimum sample size required so that the chance that the sample mean mass of raisins is less than 53 g will be no more than 2%.   (3 marks)

Question 16   **(7 marks)**

A particle travels in a straight line so that its displacement $x(t)$ cm at time t seconds, relative to fixed point O, satisfies the equation $\dfrac{d^2x}{dt^2} = -k^2x$. Initially it has a displacement of 26 cm and is moving away from O. It moves with a period of 10 seconds and an amplitude of 80 cm.

(a)   Determine a suitable function for $x(t)$.   (3 marks)

(b)   Determine the speed of the particle when it has a displacement of 60 cm.   (2 marks)

(c)   Determine the distance travelled by the particle in the first 5 seconds.   (2 marks)

Question 17   **(7 marks)**

A water tank, initially empty, is in the form of an inverted right cone of radius 4 m and depth 5 m. Water is flowing into the tank at a steady rate of 0.05 m^3 per minute but leaking out at a rate of $0.002h^2$ m^3 per minute, where h is the depth of water in the tank.

(a)   Determine the rate at which the depth of water is increasing in the tank when the depth of water reaches 3 m.   **(5 marks)**

(b)   Explain whether the tank will ever overflow.   **(2 marks)**

Question 18   **(6 marks)**

Particle A moves with velocity vector $\mathbf{v}(t) = 4t\mathbf{i} - 2t\mathbf{j} - 3\mathbf{k}$ ms^{-1}, where t is the time in seconds and $t \geq 0$. Initially, the particle has position vector $\mathbf{r} = 3\mathbf{i} + \mathbf{j} + 7\mathbf{k}$.

(a)   Determine $\mathbf{r}(t)$, the position vector of A at time t.   **(2 marks)**

A second particle B moves with constant velocity vector $3\mathbf{i} - 8\mathbf{j} - 2\mathbf{k}$ and has initial position vector $20\mathbf{i} + 25\mathbf{j} + 5\mathbf{k}$.

(b)   Determine if the paths of the particles cross and if so, whether they meet.   **(4 marks)**
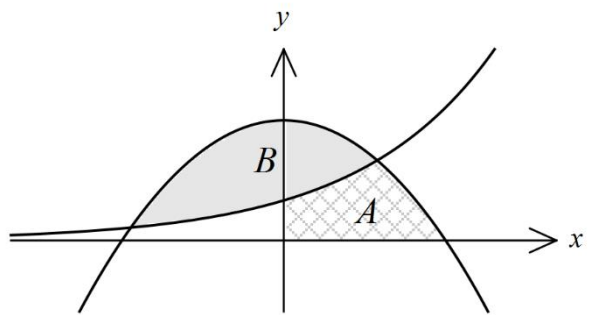
Question 19 **(7 marks)**

Let $f(x) = 4 - x^2$ and $g(x) = 3^x$.

The diagram, not to scale, shows
the graphs of $y = f(x)$ and $y = g(x)$.

Region A, in the first quadrant, is
bounded by the x-axis, the y-axis
and the two curves.

Region B, shaded, is bounded by
the two curves.

(a) Determine the area of region A. (3 marks)

(b) Determine the volume of the solid generated when region B is rotated about the horizontal
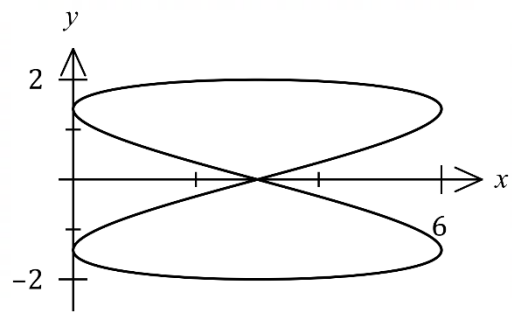line $y = -2$. (4 marks)

Question 20   **(7 marks)**

The path of a particle with position vector $\mathbf{r}(t)$ is shown in the diagram, where t is the time in seconds since motion began and

$$\mathbf{r}(t) = \begin{pmatrix} 3 - 3\sin(2t) \\ 2\sin(t) \end{pmatrix} \text{ cm}.$$

(a) State the time at which the particle first touches the y-axis.   **(1 mark)**

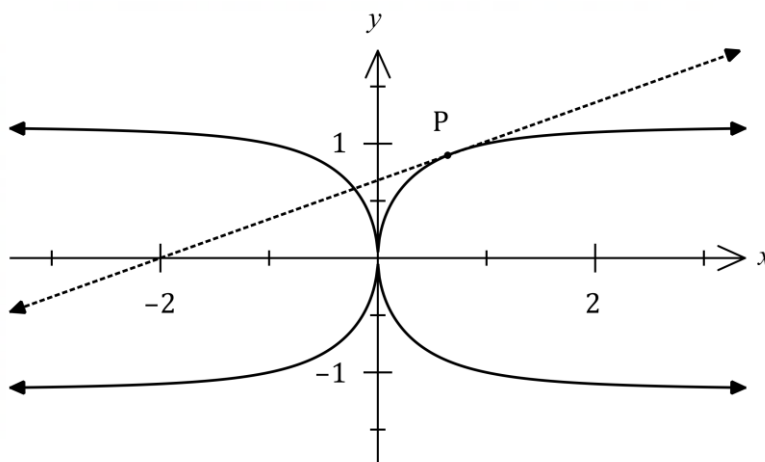(b) Determine the Cartesian equation for the path of the particle.   **(3 marks)**

(c) Determine the length of one circuit of the path.   **(3 marks)**

Question 21 (7 marks)

The graph of the relationship $y^2(3x^2 + y^2) = 4x^2$ is shown below, together with the tangent to the curve at P that passes through the point $(-2, 0)$.

(a) Use implicit differentiation to obtain an expression for $\dfrac{dy}{dx}$. (3 marks)

(b) Determine the slope of the curve at the point $(1, 1)$. (1 mark)

(c)   Deduce that the x-coordinate of P is a solution to the equation $9x^3 - 2x^2 + 8x - 8 = 0$.

(3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____